Product Rule

$$\frac{d}{dx}uv = u\frac{dv}{dx} + \mathrm{v}\frac{du}{dx}$$

The product rule is used to differentiate an expression where terms are multiplied together. This formula means that if we differentiate the value $u \times v$ then that will be equal to u ×(derivative of v) + v ×(derivative of u)[1]

For example, if we have the expression $y = (x + 5)(2x - 3)$ and we want to differentiate this without expanding the brackets.

We can say that $u = (x + 5)$ and $v = (2x - 3)$.
So

$$\frac{d}{dx}(x + 5)(x - 3) = (x + 5)\frac{d}{dx}(2x - 3) + (2x - 3)\frac{d}{dx}(x + 5)$$
$$\frac{d}{dx}(x + 5)(x - 3) = (x + 5) \times 2 + (2x - 3) \times 1$$
$$\frac{d}{dx}(x + 5)(x - 3) = 2x + 10 + 2x - 3$$
$$\frac{d}{dx}(x + 5)(x - 3) = 4x + 7$$

Proof

Let $y = uv$ where u and v are functions of x. If we increase x by δx (δx is a very small amount), then u and v increase by δu and δv and so y increases by δy
This means that

$$y + \delta y = (u + \delta u)(v + \delta v)$$

Expanding the brackets

$$y + \delta y = uv + u\delta v + v\delta u + \delta u\delta v$$

But we know $y = uv$, so subtracting that

$$(y + \delta y) - y = (uv + u\delta v + v\delta u + \delta u\delta v) - uv$$

We get

$$\delta y = u\delta v + v\delta u + \delta u\delta v$$

'Dividing' both sides by δx

$$\frac{\delta y}{\delta x} = u\frac{\delta v}{\delta x} + v\frac{\delta u}{\delta x} + \delta u\frac{\delta v}{\delta x}$$

As $\delta x \to 0$ and $\delta u \to 0$
and

$$\frac{\delta y}{\delta x}, \frac{\delta v}{\delta x}, \frac{\delta u}{\delta x} \to \frac{dy}{dx}, \frac{dv}{dx}, \frac{du}{dx}$$

So

$$\frac{d}{dx}uv = u\frac{dv}{dx} + \mathrm{v}\frac{du}{dx}$$

[1] The derivative is the expression we get as the result of differentiation

See also

- Quotient Rule

References

Turner, L. K. (1976). *Advanced Mathematics – Book One*. London: Longman. pp.113-115.